Statistical Learning Theory and SVM

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duration : 1h30
Electronic devices are not allowed, all paper documents are allowed

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Warning ! All answers have to be justified. Do never answer by only yes or no.

1 Problem definition

A Koala robot (see figure 1 on the right) lives in a maze (see figure 1 on the left). The width and length of the robot are 0.30m. On the robot, the speed values v_g and v_d of the respective left and right central wheels are continuously measured. Each speed value belongs to $[-1, 1]$, 1 being the maximal forward speed, -1 being the maximal backward speed, and 0 being the null speed. The robot is endowed with 16 infrared proximity sensors, $r_1, \cdots, r_{16}$, positioned at its periphery, each providing a value in $[0, 1]$. The sensors "see" the obstacles near their axis (the sensor axes are represented by the lines in figure 1). The value 0 means that no obstacle is visible in the axis direction (the obstacle, if any, is too far), and 1 means the the obstacle is in contact with the sensor. For example, on the figure, $r_8 = r_9 = 0$, $r_{13} = 0.7$, $r_{15} = 0.6$, $r_3 = r_4 = .05$.

An operator controls the robot from the video stream, applying $((a_g, a_d) \in [-1, 1]^2$, which are respective left wheel and right wheel accelerations. Accelerations modify the wheel speeds, but that speeds are limited within the $[-1, 1]$ range. Last, a supplementary emergency stop command exists. It blocks instantaneously the wheels.

2 Collision alert

We observe an operator who drives the robot in the maze, exploring all the maze. The operator is an expert, he knows how to drive (setting (a_g, a_d)) so that the robot goes as fast as possible, without bumping into the walls.

Every 500ms, $s^t = (v_d^t, v_g^t, r_1^t, r_2^t, \cdots, r_{16}^t)^T$ is sampled. The operator is recorded for 10 minutes, so that 1200 samples are obtained and stored into some database S.

$\boxed{\textbf{Question 2.1} \text{ (2 points)}}$: Without any pre-processing on S, and using one SVM presented during the lecture, describe a method allowing to set up an emergency stop procedure. This procedure have to trigger the emergency stop command when the robot is expected to be about to hit a wall. Explain the choice of your SVM, and detail the tuning of the parameters involved.

We decide to apply some pre-processing of the database S, thanks to two 18×18 matrices. These are the following ones, for which only non null coefficients are shown.

$$M = \begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & & & \\ & & 0.6 & 0.2 & & & & 0.2 \\ & & 0.2 & 0.6 & 0.2 & & & \\ & & & 0.2 & 0.6 & 0.2 & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & 0.2 & 0.6 & 0.2 \\ & & 0.2 & & & & 0.2 & 0.6 \end{bmatrix}, \ N = \begin{bmatrix} \frac{1}{2} & & & & & \\ & \frac{1}{2} & & & & \\ & & \frac{1}{16} & & & \\ & & & \frac{1}{16} & & \\ & & & & \ddots & \\ & & & & & \frac{1}{16} \end{bmatrix}, \ s = \begin{pmatrix} v_d \\ v_g \\ r_1 \\ r_2 \\ \vdots \\ r_{16} \end{pmatrix}$$

The pre-processing consists in considering $s' = NMs$ for each observation s.

$\boxed{\textbf{Question 2.2} \text{ (1 point)}}$: What is the role of M ?

$\boxed{\textbf{Question 2.3} \text{ (1 point)}}$: What is the role of N ?

Question 2.4 (2 point) : Let $k(s, z)$ be some kernel in $\mathbb{R}^{18}$. Prove that $k'(s, z) = k(NMs, NMz)$ is a kernel.

Question 2.5 (1 point) : Would the performances of the emergency stop procedure you have set up at the beginning of section 2 have been better if you had used k' instead of k ?

3 Localization

We use the same experimental setup ad previously, but, thanks to a video device placed on the ceiling of the room, we have at each time instance at our disposal the measure of the position (x, y) (in meters) of the robot on the ground and a measure $\alpha \in [0, 2\pi[$ of its orientation. In all the current section, *the maze is the one in figure 2-a.* Let us store in a database the samples $s^t = (r_1^t, r_2^t, \cdots, r_{16}^t, x^t, y^t, \alpha^t)$ obtained when an operator is visiting the whole maze.

Question 3.6 (2 point) : Describe the use of a SVM presented during the lecture, relying on a kernel k, allowing for the prediction of the position as well as the orientation of the robot from the infrared sensor data.

Question 3.7 (1 point) : What is the problem encountered for α ? What are its consequences ?

I use now the usual dot product as a kernel k. I obtain for the prediction of x an empirical risk that is $R_{\text{emp}} = 0.09$.

Question 3.8 (1 point) : What does $R_{\text{emp}} = 0.09$ mean ?

Question 3.9 (1 point) : What could you say about the real risk value ?

I choose now k to be a Gaussian kernel with parameter σ.

Question 3.10 (1 point) : How can I tune σ ?

Let us consider the same questions as for the linear kernel, that are :

Question 3.11 (1 point) : What does $R_{\text{emp}} = 0.09$ mean ?

Question 3.12 (1 point) : What could you say about the real risk value ?

You have now at your disposal for addressing our problem (figure 2-a) all available supervised learning algorithm in the world.

Question 3.13 (1 point) : Could you have a very low empirical risk with one of them ?

Question 3.14 (1 point) : Could you have a very low real risk with one of them ?.

4 Park the robot

We want to learn to the robot how to park from the infrared data (see figure 2-b). The robot is initially placed at a random position in the dotted lined half-circle, and an operator who can see the robot drives it so that it gets parked (at the dotted line position sketched on the bottom of the figure). We record several trajectories, starting from random positions in the half-circle. We collect the points $(r_1, r_2, \cdots, r_{16}, v_g, v_d, a_g, a_d)$ of these trajectories, the values (a_d, a_g) being the operator command.

Question 4.15 (2 points) : From the collected data and your knowledge about SVMs, propose the setting up of a procedures that would enable the robot to park autonomously when it is placed in the half-circle. Describe the implementation of the procedure.

Question 4.16 (1 point) : How would you determine the performance of that procedure ?

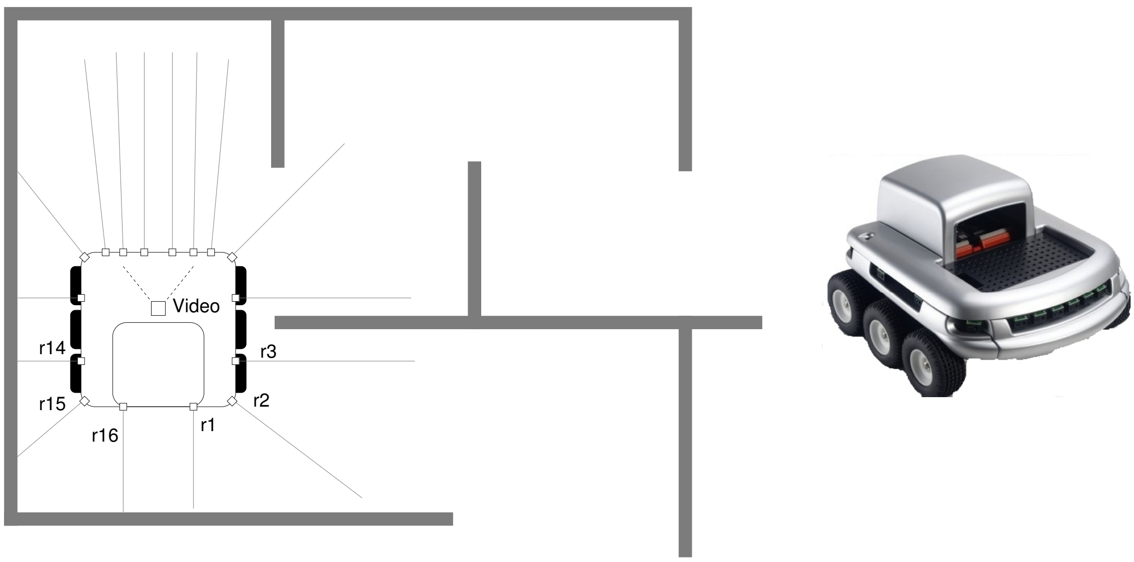

FIGURE 1 – The Koala robot in its maze

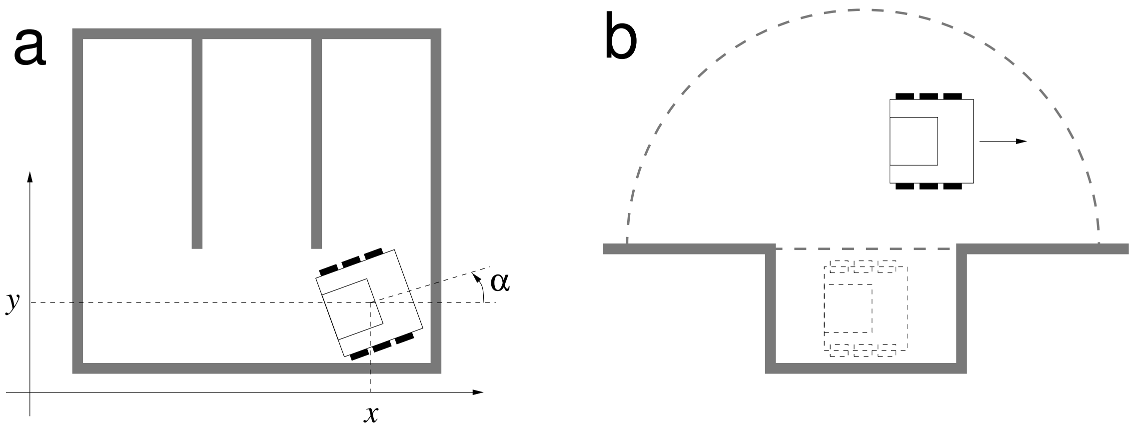

FIGURE 2 – a : robot localization. b : park the robot.